MATH 105A and 110A Review: Elementary matrices and row operations

1. Identify which matrices are in echelon form, reduced echelon form, or neither.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution: Since the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

has a row of zeros above a a row of non-zeros, then the matrix is not in echelon form. Hence, it is not in reduced echelon form.

The matrix

$$\begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is in echelon form. But it is not in reduced echelon form because the leading entry in row 1 is not 1. Additionally, we would need all entries above the leading entry of row 3 to be zero.

The matrix

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is not in echelon form because the entries below the leading entry of row 3 are not zero.

The matrix

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

is in both echelon form and reduced row echelon form.

2. Reduce A to reduced echelon form.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 1 \end{bmatrix}.$$

Solution: We have

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 1 \end{bmatrix} R_2 - 2R_1 \to R_2 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} R_3 - R_1 \to R_3 \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & -2 & -2 \end{bmatrix}.$$

Continuing

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & -2 & -2 \end{bmatrix} R_2 \leftrightarrow R_3 \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} R_1 + R_2 \to R_1 \begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} 1 - /2 \cdot R_2 \to R_2 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

which is in reduced echelon form.

3. Carry out row operation $R_2 + 3R_1 \to R_2$ on matrix A using an elementary matrix, where

$$A = \begin{bmatrix} 1/3 & 1 \\ 1 & 1 \end{bmatrix}.$$

Solution: We first obtain the corresponding elementary matrix by applying the row operation $R_2 + 3R_1 \to R_2$ on the identity matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} R_2 + 3R_1 \to R_2 \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}.$$

Hence,

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1/3 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 1 \\ 2 & 4 \end{bmatrix}.$$